RWTH RHEINISCH-WESTFÄLISCHE TECHNISCHE HOCHSCHULE AACHEN

LEHRSTUHL FÜR INFORMATIK II

RWTH Aachen · D-52056 Aachen · GERMANY

Winter term 2005/06

Prof. Dr. Ir. J.-P. Katoen

Exam in *Model Checking*

March 31, 2006

Family name: ______________________________

First name: ______________________________

Student number: ______________________________

Field of study:
☐ **Software Systems Engineering**
☐ **Informatik (Diplom)**
☐ **Others:** ______________________________

Please note the following hints:

- Keep your student id card and a passport ready.
- The only allowed materials are
  - a copy of the lecture notes and
  - a copy of the lecture slides.

  No other materials (i.a. exercises, solutions, handwritten notes) are admitted.
- This test should have five pages (including this cover sheet).
- Write your name and student number on every sheet.
- Also use the back side of the pages if needed.
- Write with blue or black ink; do not use a pencil.
- Any attempt at deception leads to failure for this exam, even if it is detected only later.
- The editing time is **90 minutes**.

Question	Possible	Received
1	10	
2	10	
3	10	
4	10	
Total	40	
Grade		

Question 1 (10 points)

Let $AP = \{a\}$ and $\varphi = (a \wedge \bigcirc a) \mathsf{U} \neg a$ an LTL–formula over AP.

(a) Compute all elementary sets with respect to φ.
Hint: There are five elementary sets.

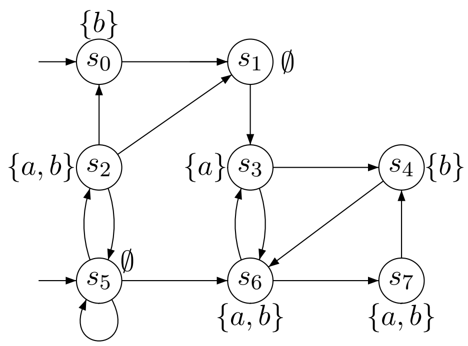
(b) Construct the GNBA $\mathcal{G}_\varphi$ according to the algorithm from the lecture such that $\mathcal{L}_\omega(\mathcal{G}_\varphi) = Words(\varphi)$.

Question 2 (10 points)

Consider the CTL*-formula (over $AP = \{a, b\}$)

$$\Phi = \forall\Diamond\Box\exists\bigcirc\big(a \mathsf{U} \exists\Box b\big)$$

and the transition system TS outlined below:



Apply the CTL* Model Checking Algorithm to compute $Sat(\Phi)$ and decide whether $TS \models \Phi$.
Hint: You may infer the satisfaction sets for LTL formulas directly.

> **Note:** In the lecture notes, there is a typo in the algorithm; the correct algorithm is given below where the missing negation symbol is highlighted.

Algorithm 1 CTL* model checking algorithm (basic idea)

Require: finite transition system TS with initial states I, and CTL*-formula Φ
Ensure: $I \subseteq Sat(\Phi)$

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for all $i \leq |\,\Phi\,|$ **do**
  for all $\Psi \in Sub(\Phi)$ with $|\,\Psi\,| = i$ **do**
    switch(Ψ):

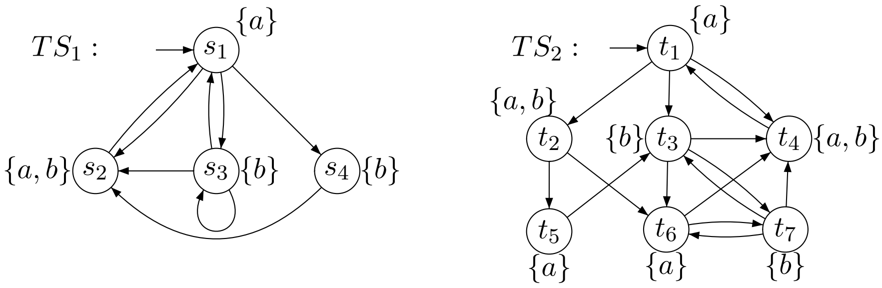
true	:	$Sat(\Psi) := S$;
a	:	$Sat(\Psi) := \{s \in S \mid a \in L(s)\}$;
$a_1 \wedge a_2$	:	$Sat(\Psi) := Sat(a_1) \cap Sat(a_2)$;
$\neg a$	:	$Sat(\Psi) := S \setminus Sat(a)$;
$\exists\varphi$	:	determine $Sat_{\mathrm{LTL}}(\varphi)$ by means of an LTL model-checker;
	:	$Sat(\Psi) :=$ $S \setminus Sat_{\mathrm{LTL}}(\neg\varphi)$ (* correction *)

    end switch
    $AP := AP \cup \{a_\Psi\}$; (* introduce fresh atomic proposition *)
    replace Ψ with a_Ψ
    forall $s \in Sat(\Psi)$ **do** $L(s) := L(s) \cup \{a_\Psi\}$; **od**
  end for
end for
return $I \subseteq Sat(\Phi)$

Question 3

(10 points)

Consider the following transition systems TS_1 and TS_2:



(a) Decide whether $TS_1 \sim TS_2$. Explain your answer formally.

(b) Decide whether $TS_1 \simeq TS_2$. Explain your answer formally.

Question 4 (10 points)

Let $AP = \{a, b\}$.

(a) P_1 denotes the linear time property that consists of all infinite words $\sigma = A_0A_1A_2\cdots \in \left(2^{AP}\right)^\omega$ such that there exists $n \geq 0$ with

$$\forall j < n.\ A_j = \emptyset \quad \wedge \quad A_n = \{a\} \quad \wedge \quad \forall k > n.\ \ (A_k = \{a\} \Rightarrow A_{k+1} = \{b\})\,.$$

(i) Give an ω–regular expression for P_1.

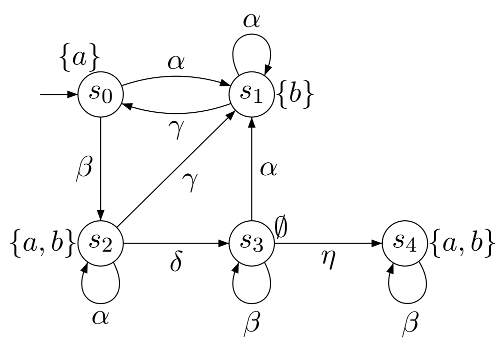
(ii) Apply the decomposition theorem and give ω–regular expressions for P_{safe} and P_{live}.

(iii) Justify that P_{live} is a liveness and that P_{safe} is a safety property.

(b) Now let P_2 denote the set of traces of the form $\sigma = A_0A_1A_2\cdots \in \left(2^{AP}\right)^\omega$ such that

$$\overset{\infty}{\exists}\, k.\ A_k = \{a, b\} \quad \wedge \quad \exists n \geq 0.\ \forall k > n.\ \big(a \in A_k \Rightarrow b \in A_{k+1}\big).$$

Consider the following transition system TS:



Consider the following fairness assumptions:

(i) $\mathcal{F}_1 = \Big(\big\{\{\alpha\}\big\}, \big\{\{\beta\}, \{\delta, \gamma\}, \{\eta\}\big\}, \emptyset\Big)$. Decide whether $TS \models_{\mathcal{F}_1} P_2$.

(ii) $\mathcal{F}_2 = \Big(\big\{\{\alpha\}\big\}, \big\{\{\beta\}, \{\gamma\}\big\}, \big\{\{\eta\}\big\}\Big)$. Decide whether $TS \models_{\mathcal{F}_2} P_2$.

Justify your answers!