

Bernoulli-Ungl. $(1+x)^n \geq 1+nx$
 $e^{\ln x} = x \quad \ln(e^x) = x$

$$(x+y)^n = \sum_{k=0}^n \binom{n}{k} x^{n-k} y^k$$

$$ar \sinh = \ln(x + \sqrt{x^2 + 1})$$

$$ar \cosh = \ln(x + \sqrt{x^2 - 1})$$

$$\ln(xy) = \ln(x) + \ln(y)$$

Mittelwertsatz $f'(x_0) = \frac{f(b)-f(a)}{b-a}$

$$\sum_{i=0}^{\infty} \frac{x^i}{i!} = e^x$$

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}$$

Taylor: $T_n(f; x) := \sum_{k=0}^n \frac{f^{(k)}(a)}{k!} (x-a)^k$

Potenzgesetze

$$a^p \cdot a^r = a^{p+r} \quad a^p \cdot b^p = (ab)^p$$

$$\frac{a^p}{a^r} = a^{p-r} \quad \frac{a^p}{b^p} = \left(\frac{a}{b}\right)^p$$

$$(a^p)^r = a^{pr}$$

Differentiation

$$(f+g)' = f' + g'$$

Produktregel: $(f \cdot g)' = f' \cdot g + f \cdot g'$

Quotientenregel: $\left(\frac{f}{g}\right)' = \frac{f' \cdot g - f \cdot g'}{g^2}$

Kettenregel: $(f \circ g)' = g' \cdot f'(g(x))$

$$(f^{-1})' = \frac{1}{f'(f^{-1}(x))}$$

$$\frac{2}{3a}(ax+b)^{\frac{3}{2}} \quad ()' = \sqrt{ax+b}$$

$$\frac{a^x}{\ln a} \quad ()' = a^x$$

$$x(\ln(x)-1) \quad ()' = \ln x$$

$$\frac{1}{x^n} \quad ()' = -\frac{n}{x^{n+1}}$$

$$\sqrt{x} \quad ()' = \frac{1}{2\sqrt{x}}$$

$$\sqrt[n]{x} \quad ()' = \frac{1}{n\sqrt[n]{x^{n-1}}}$$

$$\ln x \quad ()' = \frac{1}{x}$$

$$a^x \quad ()' = a^x \ln a$$

$$\tan x \quad ()' = \frac{1}{\cos^2 x} = 1 + \tan^2 x$$

$$\cot x \quad ()' = -\frac{1}{\sin^2 x} = -1 - \cot^2 x$$

$$\arcsin x \quad ()' = \frac{1}{\sqrt{1-x^2}}$$

$$\arccos x \quad ()' = -\frac{1}{\sqrt{1-x^2}}$$

$$\arctan x \quad ()' = \frac{1}{1+x^2}$$

$$\operatorname{arccot} x \quad ()' = -\frac{1}{1+x^2}$$

$$\sinh x \quad ()' = \cosh x$$

$$\cosh x \quad ()' = \sinh x$$

$$\tanh x \quad ()' = \frac{1}{\cosh^2 x} = 1 - \tanh^2 x$$

$$\operatorname{coth} x \quad ()' = -\frac{1}{\sinh^2 x} = 1 - \operatorname{coth}^2 x$$

$$ar \sinh x \quad ()' = \frac{1}{\sqrt{x^2+1}}$$

$$ar \cosh x \quad ()' = \frac{1}{\sqrt{x^2-1}}$$

$$ar \tanh x \quad ()' = \frac{1}{1-x^2}$$

$$ar \operatorname{coth} x \quad ()' = \frac{1}{1-x^2}$$

Integration

partielle Integration $\int uv' = uv - \int u'v$

$$\int (ax+b)^n = \frac{(ax+b)^{n+1}}{a(n+1)} = \frac{1}{a} \ln|ax+b|$$

$$\int \ln x = x \ln x - x \quad \forall x > 0$$

$$\int (\ln x)^n = x(\ln x)^n - n \int (\ln x)^{n-1}$$

$$\int \frac{1}{x} \ln x = \frac{1}{2} (\ln x)^2$$

$$\int \frac{1}{x \ln x} = \ln|\ln x|$$

$$\int \frac{f'(x)}{f(x)} = \ln|f(x)|$$

$$\int \frac{1}{ax^2+bx+c} = \frac{2}{\sqrt{4ac-b^2}} \arctan \frac{2ax+b}{\sqrt{4ac-b^2}}$$

$$\int \frac{1}{x^2+a^2} = \frac{1}{a} \arctan \frac{x}{a}$$

uneigentliche Integrale

$$\int_a^{\infty} f(x) dx := \lim_{b \rightarrow \infty} \int_a^b f(x) dx$$

Konvergenz

Wurzel-Krit. $\lim_{n \rightarrow \infty} \sqrt[n]{|u_n|} < 1$

Quotienten-Krit. $\lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right| < 1$

unbestimmbar = 1, divergent > 1

Leibnitz-Kriterium:

alternierend, konvergent gegen 0, nicht-alternierender Teil muß gegen 0 konvergieren, monoton fallend

Konvergenzradius $\lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right| = \frac{1}{R}$

$$\lim_{n \rightarrow \infty} \sqrt[n]{|u_n|} = \frac{1}{R}$$

$$\lim_{n \rightarrow \infty} \frac{n^n e^{-n} \sqrt{2\pi n}}{n!} = \lim_{n \rightarrow \infty} \frac{n}{\sqrt[n]{n!}} = 1$$

Additionstheoreme etc.

$$\cos(x+y) = \cos x \cdot \cos y - \sin x \cdot \sin y$$

$$\sin(x+y) = \sin x \cdot \cos y + \cos x \cdot \sin y$$

$$\cos^2 x + \sin^2 x = 1$$

$$\cosh^2 x - \sinh^2 x = 1$$

$$\cosh(-x) = \cosh(x)$$

$$\sinh(-x) = -\sinh(x)$$

$$2 \sin(x) \cos(x) = \sin(2x)$$

$$2 \sinh(x) \cosh(x) = \sinh(2x)$$

$$\cosh(x) + \sinh(x) = e^x$$

$$\cos(2x) = \cos^2(x) - \sin^2(x)$$

"a priori" Absch.: $|x_n - x^*| \leq \frac{L^n}{1-L} |x_0 - x_1|$

Besondere Funktionswerte:

	0°	30° ($\frac{\pi}{6}$)	45° ($\frac{\pi}{4}$)	60° ($\frac{\pi}{3}$)	90° ($\frac{\pi}{2}$)
sin	0	$\frac{1}{2}$	$\frac{1}{2}\sqrt{2}$	$\frac{1}{2}\sqrt{3}$	1
cos	1	$\frac{1}{2}\sqrt{3}$	$\frac{1}{2}\sqrt{2}$	$\frac{1}{2}$	0
tan	0	$\frac{1}{3}\sqrt{3}$	1	$\sqrt{3}$	-